Early Mathematics Guidance

Aims

This document aims to outline some of the key concepts in early mathematics, so that adults can support children's learning of number and place value, as well as basic addition and subtraction.

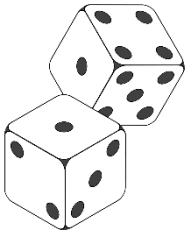
Counting – the one-to-one principle: we assign one number name to one object

When we first teach children to count, we show them that they need to tag each object with a number word. We give each object only one number word. We only use each number word once.

- **Activities**: counting things of different sizes, shapes and materials.
- **Look out:** some children count objects more than once or miss out objects that need to be counted.
- **Support:** line up the objects to be counted and show the children how you can count by placing your finger on an object and saying a number word, then moving on to another object and doing the same. Then help them to do the same.
- **Key question:** Can you put the objects in a line and count one at a time?

Subitising – fast recognition of up to 4 items

Subitising is recognising how many things are in a group without having to count them one by one. Children need opportunities to see regular arrangements of small quantities, e.g., a dice face, structured manipulatives, etc., and be encouraged to say the quantity represented. Children also need opportunities to recognise small amounts when they are not in the 'regular' arrangement, e.g., small handfuls of objects.



- **Activities**: see small groups of objects and say how many there are, you can use dot cards, or dice or dominoes for example.
- **Look out:** some children will still try and count the dots, but we want them to be able to 'see the amount' without counting. Some children may need more help with this when objects are scattered, different sizes or represented in unfamiliar ways.
- **Support:** start with small quantities, one or two to begin with, and try a 'show and hide' routine – where children tell you how many they saw before you hid them.
- **Key question:** What do you see and how do you see it?

Counting – the stable-order principle: we say the numbers in a certain order

Children need to know that when we count, we say the words in a certain order. They will need to learn that these number names follow a sequence which we can say forwards, when counting on, or backwards, when counting back.

- **Activities**: number rhymes and counting books can help children to learn the order of the number names that we use. Make sure children are familiar with counting forwards before introducing counting backwards. When children become more confident, try starting on different numbers.
- **Look out:** some children find it difficult to cross tens boundaries or skip numbers out.
- **Support:** using a number track, which shows the number name alongside/beneath a picture of the quantity can help to demonstrate that the order doesn't change; and it can reduce the demands on working memory when counting back.
- **Key question:** Which number comes next after …?

Cardinality: the last number we say is how many there are altogether

The cardinal value is how many there are. They are taught to recognise that, when we count or give out a specific number of things, the number that we stop on is the amount we have so far.

- **Activities**: playing dice games involving a number track or involving collecting or giving out several things
- **Look out:** some children might always count the whole group, rather than counting out the specific number
- **Support:** provide opportunities for counting out or 'giving' a specific number of things from a larger group, not just to count how many things there are altogether. This is to support them in focusing on the 'stopping number' which gives the cardinal value.
- **Key question:** How many are there altogether?

Counting – the abstraction principle: we can count things we can't see or touch

This is when children learn that we can count all sorts of things, even things we can't see or touch – like, sounds, movements or things we can visualise or imagine.

- **Activities**: begin by counting things that are out of reach, then try counting things passing by the window, before moving on to counting sounds.
- **Look out:** some children may find it hard to visualise several objects, or struggle with counting on accurately.
- **Support:** we can keep track of the number of things we have counted by making marks (or keeping a tally), counting on our fingers, or moving a counter along a number track.
- **Key question:** How many did you see/hear/imagine?

Counting – the order-irrelevance principle – we can count objects in any order

Children learn that it doesn't matter what order we count a group of objects in, as long as we only count each object once. There will still be the same number of objects.

- **Activities**: encourage children to count a group of items in different ways, starting with different objects – either left to right, right to left, top to bottom, bottom to top, in a random order.
- **Look out:** make sure children don't count objects more than once but do count every object.
- **Support:** use a number track or ten frame to support counting, placing objects on the spaces one at a time. Then repeat this in a different order to show that the total remains the same.
- **Key question:** Can you count them in a different order? What do you notice?

Recognising and naming digits – understanding what numerals mean

Children are taught that we use specific numerals to stand for a quantity of objects or items. They can match a number of things to the number symbol.

- **Activities**: find the right number of counters to match the digit shown on a card, or 'tidy up' labels to show how many things belong in each container and use this for checking when tidying away
- **Look out:** children may confuse similar-looking digits, e.g., 6 and 9, 1 and 7, or 3 and 8.
- **Support:** use a pictorial representation of the amount next to, or underneath, the digit. You could use dice, or dominoes or *Numicon*.

- **Key question:** Can you show me this many?

Conservation – the quantity doesn't change when I reorder or rearrange a group of items or objects

This is linked to the last idea. This time children learn to understand that the number of objects in a group doesn't change when the objects are rearranged or reordered.

- **Activities**: rearrange a group of objects, showing that nothing has been added or taken away; ask the children to count the number of objects in the group before and after rearrangement.
- **Look out:** children keep counting the group of objects each time, after rearrangement, even when they know that nothing has been added or removed.
- **Support:** place objects on a number track when they are counted. Show that the order they are placed on the track can vary, but the total number stays the same.
- **Key question:** Is the amount still the same now?

Matching words, digits and quantities – including using different representations

Provide children with opportunities to demonstrate that they can represent and understand numbers as words, digits and quantities. You could use a range of resources to support this, including dot patterns, *Numicon*, ten frames, dominoes etc.

- **Activities**: matching games and tasks, using dot patterns, digit cards and number word labels.
- **Look out:** children may confuse number symbols (numerals) that look alike.
- **Support:** begin with familiar quantities that children can subitise easily.
- **Key question:** Can you find the matching number, word and quantities?

Comparison – ordering and comparing numbers using the words more and less

This is where children understand that comparing numbers involves knowing which numbers are worth more or less than each other. To be able to achieve this, children need to understand cardinal values, have a sound understanding of the counting sequence; and know that when we count on, the numbers increase in value or worth (because the next number is worth one more).

A) Comparison – Inequalities: recognising when amounts are not the same as each other.

- **Activities**: find or show groups of objects that do not have the same number of things in them. Collections should vary in size, material, and colour so that the adult can draw attention to the number of things as being the most important when comparing quantity.
- **Look out:** children may confuse size and quantity.
- **Support:** begin with showing groups that are clearly not the same in number, including showing lots of small things and very few large things to highlight that it is 'How many?' we are interested in.
- **Key question:** Are these two quantities the same as each other?

B) Comparison – More than and less than: identifying which number is worth more or less than the other(s).

- **Activities**: model how to identify and label the group containing the largest number of things and show that this is 'more than' the smaller group, and (conversely) show how to identify the group containing fewer things and show that this is 'less than' the larger group. Then ask children to do the same, comparing groups of the same object/colour/size before varying the collection.
- **Look out:** children may have difficulty comparing more than two groups; or using the words *most* and *least/fewest*.

- **Support:** begin with comparing two groups and model the expected language clearly.
- **Key question:** Which has more or less? How do you know?

C) Comparison – Equality: identifying when two or more groups have the same amount or making groups equal.

- **Activities**: ask children to convert two unequal groups into two that have the same number, e.g., 'Teddy has 6 apples and Bear has 4 apples. Can we move some apples so they both have the same?'
- **Look out:** some children may not think it is equal or 'fair' if the objects aren't the same size as one another.
- **Support:** focus on the number of objects in a group, not their size, at this stage. Use non-examples to show when things are not equal as a comparison.
- **Key question:** Can you make these two groups have the same?

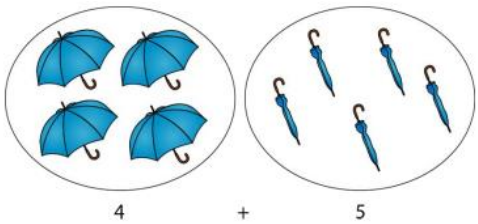
D) Comparison – Number sequences: forwards and backwards

- **Activities**: show that when we count forwards, we add one more each time, but when we count backwards we take one away each time. Singing counting rhymes or reading number books can help to show this.
- **Look out:** children may have difficulty predicting which number will come next.
- **Support:** help focus on the value of each digit to show the difference between consecutive numbers in the counting sequence.
- **Key question:** Which number do you think will come next? Can you show me?

Addition and Subtraction: Counting all – combining two amounts (aggregation) and counting all

Children first learn to find a total by counting the number of items in each group then counting all of the items to find the total. For example, children count three counters and then five counters and then find the total by counting all eight counters. Aggregation is the name given to this structure of addition.

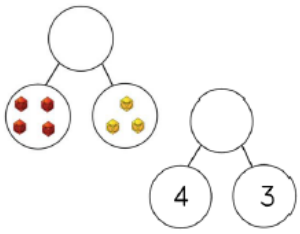
- **Activities**: provide practical opportunities for children to count real things, focussing first on totals less than 5 before moving on to totals up to and including ten
- **Look out:** some children can lose track of which items they have counted, so encourage them to line-up the items in a row and count them from left to right
- **Support:** use counting-on rhymes to help children to secure their understanding of the counting sequence
- **Representations:** children can represent their addition with counters on a number track; later they will learn to represent this using a part-whole model
- **Key question:** If we add … more, how many will we have in total?

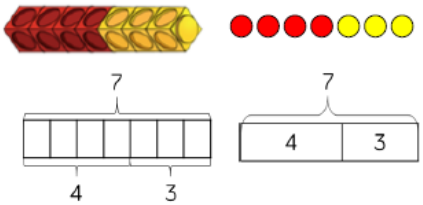


- *'There are four open umbrellas and five closed umbrellas.'*
- *'We can write this as four plus five.'*

4 + 5

- *'The 4 represents the four open umbrellas.'*
- *'The 5 represents the five closed umbrellas.'*





Images: White Rose Maths (above), NCETM (left)

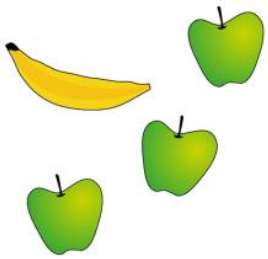
Addition and Subtraction: Composition to 5 (partitioning)

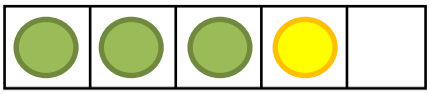
In Reception, children learn to recognise that numbers can be made up of other smaller numbers. This idea of composition is the beginning of learning 'number bonds' – ways of making one number using other numbers. At this stage teachers introduce 'part-whole' models to demonstrate that a whole amount can be made up of several parts – we call this partitioning.

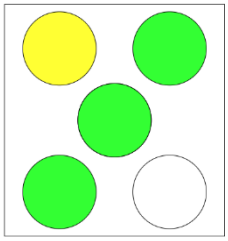
This builds on children's learning about subitising (fast recognition of numbers up to four) and seeing a number as a collection of items.

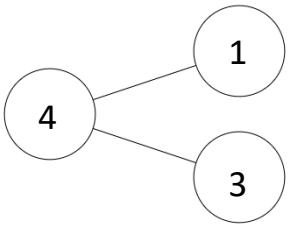
- **Activities**: provide opportunities for children to find parts within a larger set, or whole, with practical objects and then make a model of this with counters or by drawing a picture
- **Look out:** some children may think that the part can be larger than the whole
- **Support:** try making the same number in lots of different ways
- **Representations:** children can represent their addition with counters on a number track and through using a part-whole model, a five frame or a ten frame
- **Key question:** How many different ways can we use small(er) numbers to make this amount?

The images below show different ways of representing one composition for the number four:

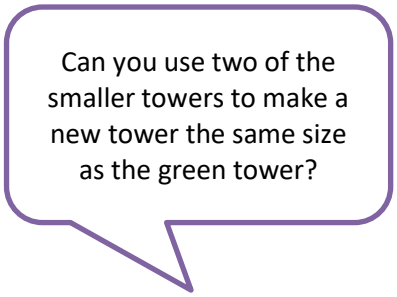








An activity like this can be used to encourage children to find different ways of making the number four:

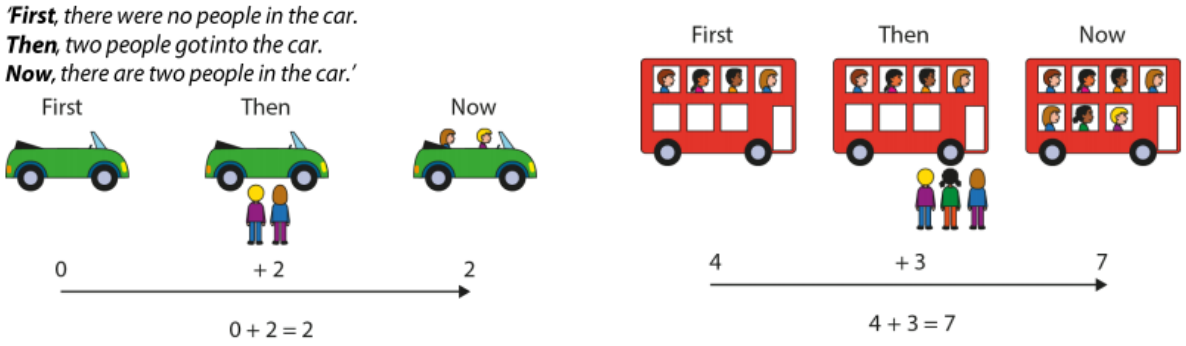


Children will explore the numbers to ten in this way, first starting with numbers up to 5 in Reception before exploring number bonds to 10 in Year 1 (this is referenced later in this document).

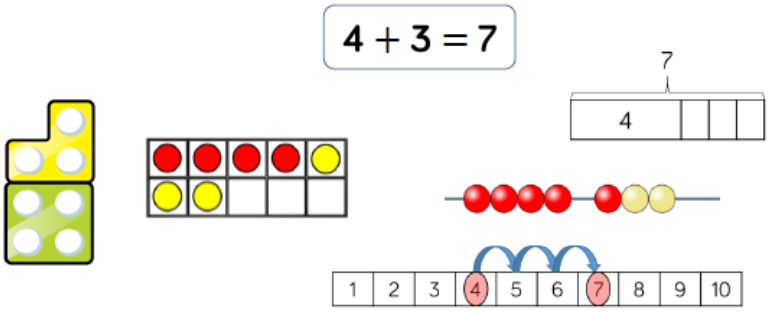
Addition and Subtraction: Counting on - combining two amounts by counting on (augmentation)

The next structure of addition that children learn about is augmentation. This involves addition by counting on. In practical situations, this type of addition can be described by a 'first…, then…, now…' story. Children learn to represent addition stories showing how one amount has been added to another.

- **Activities**: provide opportunities for children to act out and represent addition stories with a 'first…, then…, now…' structure, for example we could use the situation of *'First there were three people on the bus, then two more people got on the bus. Now there are five people on the bus.'*
- **Look out:** some children might assume that we always start with a large number and add a small number, whereas we might start with as little as 0 items and add some on
- **Support:** make sure children experience situations starting with 0 or 1
- **Representations:** children can represent their addition with counters on a number track, cubes, towers, part-whole models, five or ten frames, or number lines.
- **Key question:** If we add … and … together, how many will there be altogether?



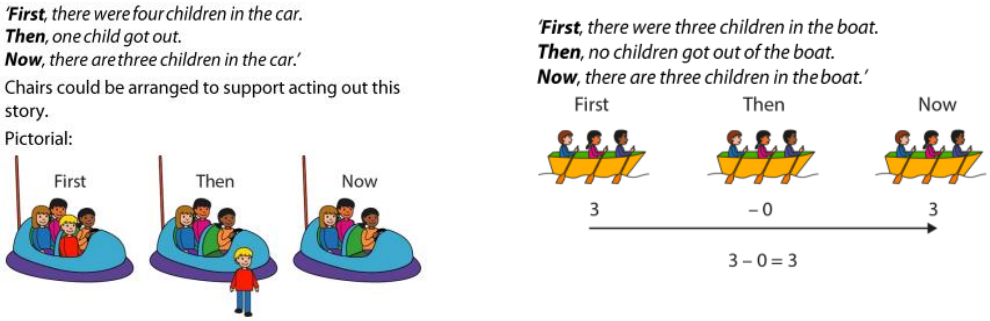
Images: White Rose Maths (below), NCETM (above)



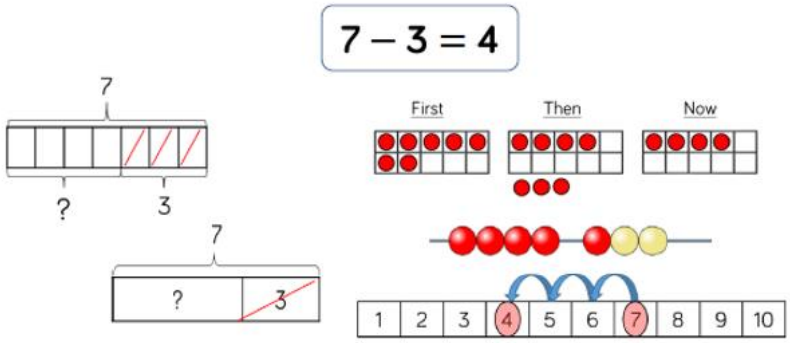
Addition and Subtraction: Counting back - subtraction by taking away or counting back (reduction)

This structure of subtraction is closely linked to the counting on (augmentation) structure for addition, but in reverse. Children learn to start with an amount and reduce this by taking away, which we can do by counting back from the number we started on. This also follows the 'first…, then…, now…' story structure children will experience when adding on to an amount.

- **Activities**: provide opportunities for children to act out and represent subtraction stories with a 'first…, then…, now…' structure, for example we could use the situation of *'First there were five birds in a tree, then two birds flew away. Now there are three birds in the tree.'*
- **Look out:** some children might assume that there will always be some left over
- **Support:** make sure children experience situations where all the objects are taken away and 0 are left
- **Representations:** children can represent their subtractions with counters on a number track, cubes, towers, part-whole models, five or ten frames, or number lines.
- **Key question:** First we had …, then … was taken away. How many are there now?



Images: White Rose Maths (below), NCETM (above)



Addition and Subtraction: One more and one less

Children learn that the next number we say is one more if we are counting on, and that it is one less if we are counting back.

- **Activities**: provide opportunities for children to represent the size of numbers and notice the value of consecutive numbers in the counting sequence, e.g., comparing 4 with 5 as well as comparing 4 with 3
- **Look out:** some children find it hard to visualise the size of numbers
- **Support:** make sure children have opportunities to represent and compare quantities of objects
- **Representations:** children can represent the counting sequence with cubes, counters and *Numicon* tiles
- **Key question:** If we add one more (or take one away) how many will there be?

Addition and Subtraction: Understanding symbols – addition (+), subtraction (-), equals (=)

Children learn that the words addition and subtraction can be represented by symbols, + and -. Children also use the symbol (=) to show that two expressions are equal, or balanced.

- **Activities**: provide opportunities for children to use symbols within expressions to match addition and subtraction stories

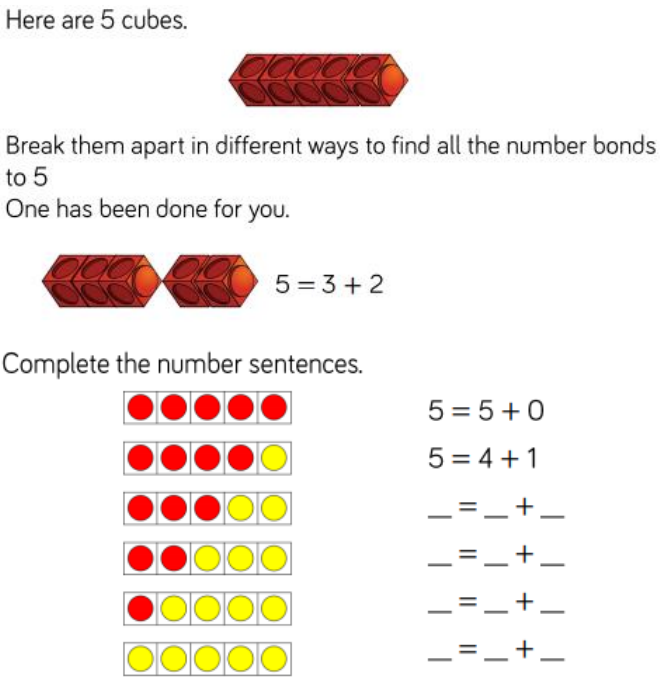
- **Look out:** some children assume that = means 'the answer is', which means they can be confused when they see number sentences such as 6 = 5 + 1, or 7 = 10 – 3.
- **Support:** make sure children understand the phrase 'is the same as' and 'is equal to' before introducing =
- **Representations:** help children to write number sentences that match various representations, including part-whole models
- **Key question:** How can you show the number story with symbols and digits?

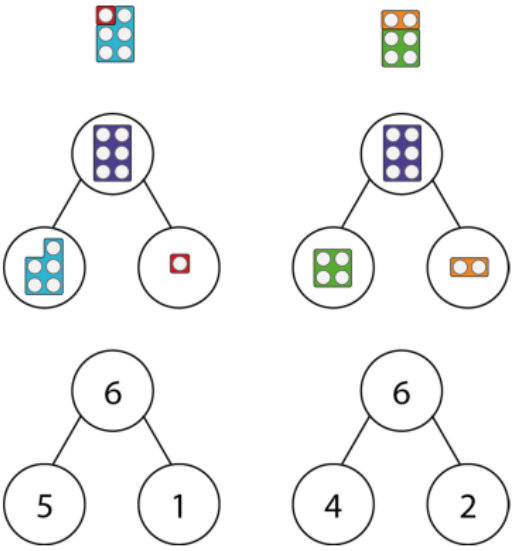
Addition and Subtraction: Number bonds to 10 – making numbers in different ways

Children build on the idea of composition within five, by learning to find and recall the different ways of making numbers up to and including ten.

- **Activities**: provide opportunities for children to partition a number in different ways, using equipment
- **Look out:** some children assume that number bonds can't include the number 0
- **Support:** make sure children understand the link between number bonds and related subtraction facts
- **Representations:** representations which show the part-whole relationship are very helpful
- **Key question:** How many ways can you break … into parts? Show how many facts you can make.

Images: White Rose Maths (left) and NCETM (right)





Supporting Resources

- NCETM – Early Years: Six Key Areas of Mathematics
  https://www.ncetm.org.uk/in-the-classroom/early-years/
- NCETM – Professional Development Materials
  https://www.ncetm.org.uk/teaching-for-mastery/mastery-materials/primary-mastery-professional-development/
- White Rose – Reception Scheme Guidance for Teachers 2020-21
  https://wrm-13b48.kxcdn.com/wp-content/uploads/2020/07/Reception-Scheme-Guidance-for-teachers-Autumn-2020.pdf
- White Rose – Calculation Policy for Addition and Subtraction, Multiplication and Division
  https://whiterosemaths.com/resources/primary-resources/primary-sols/